

How to Measure the Earth's Circumference?

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The chapter of my book, "The Enchanting Encounter with the East", compares the estimations of the Earth's circumference made by Greek, Arab, and European scholars. It sets the 'consensus' value of ancient measures to facilitate its conversion to modern equivalents.

Tags: Stadium, Arab mile, Roman mile, Eratosthenes, Posidonius, Pierre d'Ailly, Ahmad Al-Farghani, Christopher Columbus

We have seen that there was a consensus among eminent medieval academics concerning the shape of the earth. Yet, the scientific circles got involved in a heated argument about the calculation of the globe's circumference.

Back in time, the two giants of ancient Greek science endeavored to carry out this tremendous task. Both estimated the angle of elevation of heavenly bodies above the horizon at distant points presumably located on the same meridian to calculate the measure of a degree of the perimeter. They also assessed the distance between these spots by converting travel days on land (1) or at sea (2) into a valid standard of length.

The fact that each of these estimations was available in two versions shows that the scholars were unhappy with their initial scores and, being confronted with sound criticism, had to improve their performance. The margin of uncertainty only grows if we make allowances for the inherent inconsistency between antique and modern units of measurement.

Modern science rates the equatorial circumference of the earth at 40,075 kilometers where each meridian degree stands for 111.319 kilometers. (3)

In the Greek-speaking world, a conventional unit of length to estimate distances both on land and at sea was a stadium, whose value varied from region to region. The problem with this standard lies in its ambiguity: there were no fewer than five different versions and any talk about the "precision" of ancient estimations sets up a straw man argument.

Table 2: Stadia to meters conversion (4)

Stadium Name	Metric Value
Itinerary	157
Olympic	176
Attic	185
Babylonian	196
Phoenician	209

The maximum gap between the opposite values can reach 25 percent ($157:209=0.75$). The mean or “consensus” equivalent (184.6), which cuts by half the span of error in this ocean of uncertainty, approaches the Attic stadium. The same standard also allows the easy conversion from the Roman mile to the stadium (1,480 meters:8=185 meters), as reported by Strabo: “reckon, as the generality of persons...a mile at eight stadia.” (5)

By coincidence, the modern metrical value of a nautical mile—1850 meters--equals ten stadia. We will apply our “consensus” magnitude for further calculations to appreciate the nuts and bolts of mathematical geography.

The first of scientific prodigies was Eratosthenes of Cyrene, Libya, who flourished in the third century BCE and coined the term “geography”, i.e., the description of the world. In his initial estimation, he reckoned that the Earth circumference amounted to 250,000 stadia (46, 250 kilometers), a 15 percent overestimation versus modern value.

On his second attempt, the Hellenistic scholar rolled up his figures to 252,000 stadia (46,620 kilometers), probably for the sake of arithmetic convenience since his new meaning hits 700 stadia (129.5 kilometers) per latitudinal degree at the equator. Who does not adore round numbers? The second estimate had surpassed the current standard by 16 percent.

Another genius, Posidonius of Apamea, Syria, a celebrated astronomer, who lived in the first century BCE and was known for calculating the distance between the earth and the sun, disagreed with this outcome. According to his primary estimation, he set the terrestrial circumference at 240,000 stadia (44,400 kilometers). His mean figure errs only by 11 percent and considerably improves Eratosthenes’ numbers.

Unfortunately, Posidonius did not follow his sound logic and revised his original score, diminishing it to 180,000 stadia (33,300 kilometers) to reach a spectacular figure of 500 stadia (92.5 kilometers) per degree. The new value is well beyond the present-day standard, underestimating it by 17 percent.

If we compare the values obtained by the outstanding Greek savants, the score of Posidonius in his first estimation is closer to modern computation. However, in his further attempt, he made a fatal error, causing the "Blue Marble" to shrink even greater than Eratosthenes had blown it up.

Taking into consideration the passion of the Greek academics to round numbers, it's a pity that none of them tried a version of 600 stadia (111 kilometers) per degree, which would enable the champions of the exact science to come up with the trustworthy figure of 39,960 kilometers--a near hit to the modern value.

Table 3: Estimation of the Earth's Circumference by Greek Scholars (6)

Geographer	Circumference in km	The discrepancy from the modern value
Eratosthenes (250,000)	46,250	+15%
Eratosthenes (252,000)	46,620	+16%
Posidonius (240,000)	44,400	+11%
Posidonius (180,000)	33,300	-17%

In the early fifteenth century, Pierre D'Ailly, a French theologian and natural philosopher, put forward another learned opinion based on the estimate of Ahmad Al-Farghani, a Persian astronomer who thrived in the ninth century CE and belonged to the supporters of the "giant Earth". The latter assumed that the length of one longitudinal degree at the equator equaled 56.67 miles.

A famous astronomer should have used an Arab mile which is worth between 1,925 and 1,995 meters (with the "consensus value" staying at 1,960 meters). If we take the last estimate for granted, al-Farghani's degree can be converted into 111.1

kilometers which makes his circumference equal to 39,986 kilometers--a close match to the modern calculation (with a discrepancy of fewer than 90 kilometers).

However, D'Ailly made his revision in Roman miles, plummeting the value of the meridian degree to 83.872 kilometers, and coming up with the "dwarf Earth" of 30,194 kilometers (7). His estimation opened a 25 percent gap against the modern calculations.

Christopher Columbus, an involuntary discoverer of America and a self-proclaimed master of oceanic expanses, adopted this latest estimate, which drastically curtailed the circumference. He benefited from the contraction of the Earth's dimensions, arguing that the Ocean Sea was much narrower than the mainstream scholar opinion had maintained and could be crossed by contemporary sea-going vessels. By hook or by crook, he obtained the royal edict allowing him to launch his crucial venture.

Columbus confronted his figures with what he considered to be a profane concept of the "giant Earth". He also claimed, following the apocryphal composition of Esdras (8), that the dry land occupied the lion's share of our planet: six parts out of seven. He was certain that his calculations matched both the "truth" of the Scriptures and the opinion of expert mariners. Even facing the dubious results of his glorious discovery, this die-hard had never had second thoughts.

In the letter to the Spanish monarchs sent in 1503, during his last voyage, the pioneering seafarer claimed: "The world is but small: out of seven divisions of it the dry part occupies six, and the seventh is entirely covered with water... the world is not so large as vulgar opinion makes it... one degree from the equinoctial line measures fifty-six miles and two-thirds. This is a fact that one can touch with one's fingers." (9)

In another letter to the same recipient, the stubborn adventurer admits that the spiritual foundations of his mission had far eclipsed any scientific arguments: "the execution of the enterprise of the Indies was not a question of scholarship, or mathematics, or Mappa Mundi; (10) it was the fulfillment of what Esdras has said." (11)

His opponents, the advocates of the "giant Earth" conception, could cite an alternative opinion, which backed Eratosthenes's figures: "it is calculated that the circuit of the entire world is 252,000 stadia." (12)

Notes:

- (1) Herodotus implies that during a walking day an average traveler can cover 150 stadia (nearly 28 kilometers). See Herodotus Histories, 5: 54. (Indianapolis: Hackett Publishing Co., Inc., 2014), 284
- (2) The blunders in calculating maritime distances led to a wide discrepancy: Eratosthenes estimated the sea route between Rhodes and Alexandria equal to 5,000 stadia (925 kilometers), while Posidonius reduced it to 3,750 stadia (694 kilometers), opening a twenty-five percent gap. See J. Evans, The History and Practice of Ancient Astronomy. (Oxford: University Press, Inc., 1998), 64-65
- (3) E. Dawson, Weighing the World: The Quest to Measure the World. (Oxford: University Press, Inc., 2006), 7
- (4) Entry 'Stadion' (unit of length)
[http://en.wikipedia.org/wiki/Stadion_\(unit_of_length\)](http://en.wikipedia.org/wiki/Stadion_(unit_of_length))
- (5) The Geography of Strabo, 7.7.4. (London: Henry G Bohn, 1854), 495
- (6) See N. Walkup, "Eratosthenes and the Mystery of the Stade"
<http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=646&pf=1>
- (7) E. Grant, Planets, Stars, and Orbs: The Medieval Cosmos. (Cambridge: University Press, 1996), 621.
- (8) Esdras 6:42: "On the third day you commanded the waters to be gathered together in a seventh part of the earth, but you made six parts dry."
<http://www.biblegateway.com/passage/?search=2+Esdras+6%3A42&version=CEB>
- (9) R.H. Major, (ed.) Four Voyages to the New World. (Gloucester: Corinth Books, Inc.), 1978), 177-178
- (10) The so-called theological 'world map', i.e. the medieval graphic image of the habitable Earth.
- (11) A.J. Aizenstat, Pride of Obsession: The Unusual Life of Christopher Columbus. (Lincoln: iUniverse.com, Inc., 2001), 41
- (12) S. I. Sobecki, (ed.) The Sea and Englishness in the Middle Ages. (Cambridge: D.S. Brewer, 2011), 136